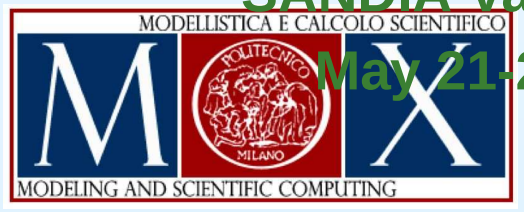
Validation Challenge Problems: Static Frame Problem

I. Babuška[a], F. Nobile[b] and R. Tempone[c]

[a] ICES, The University of Texas at Austin

[b] MOX, Politecnico di Milano, ITALY

[c] SCS and MATH, Florida State University at Tallahassee.



SANDIA Validation Challenge Workshop

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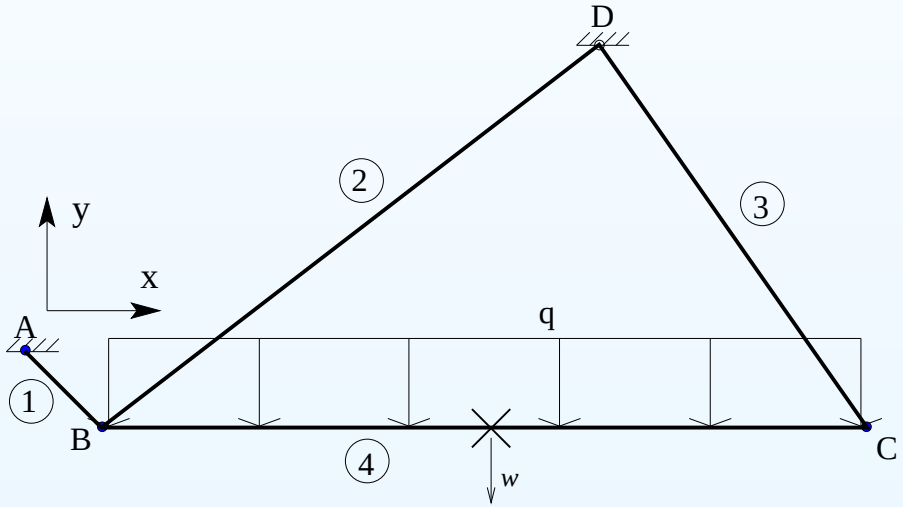
Description of the problem

PROBLEM



PREDICTION

probability of the event

$$\{w \leq 3.0 \times 10^{-3} m\}$$

w: **vertical displacement at midpoint of bar #4**

Remarks

a) **No error in measurements**

b) **Complete verification of numerical solution (no numerical error)**

Preliminary analysis

The following basic principles are assumed valid

- **Newton law**

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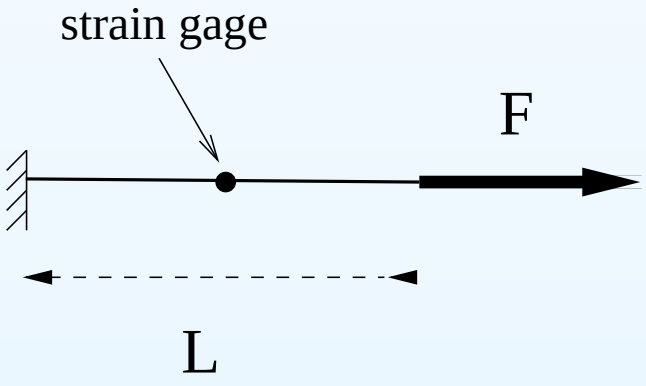
Remark: the material is a generic one (not specific engineering material). Experiments are virtual.

Calibration experiments

Goal **Determine the relation stress/elongation depending on the length of the bar: characterize Compliance or Young modulus as random fields.**



elongation measured on

- **very small length: strain gage**
- **specimen length (dog bone)**

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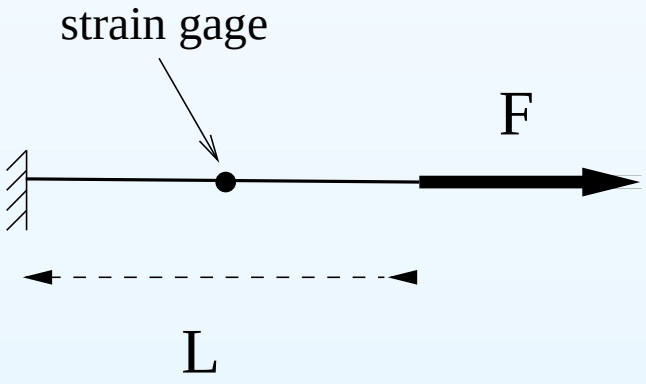


elongation measured on

- **very small length: strain gage**
- **specimen length (dog bone)**

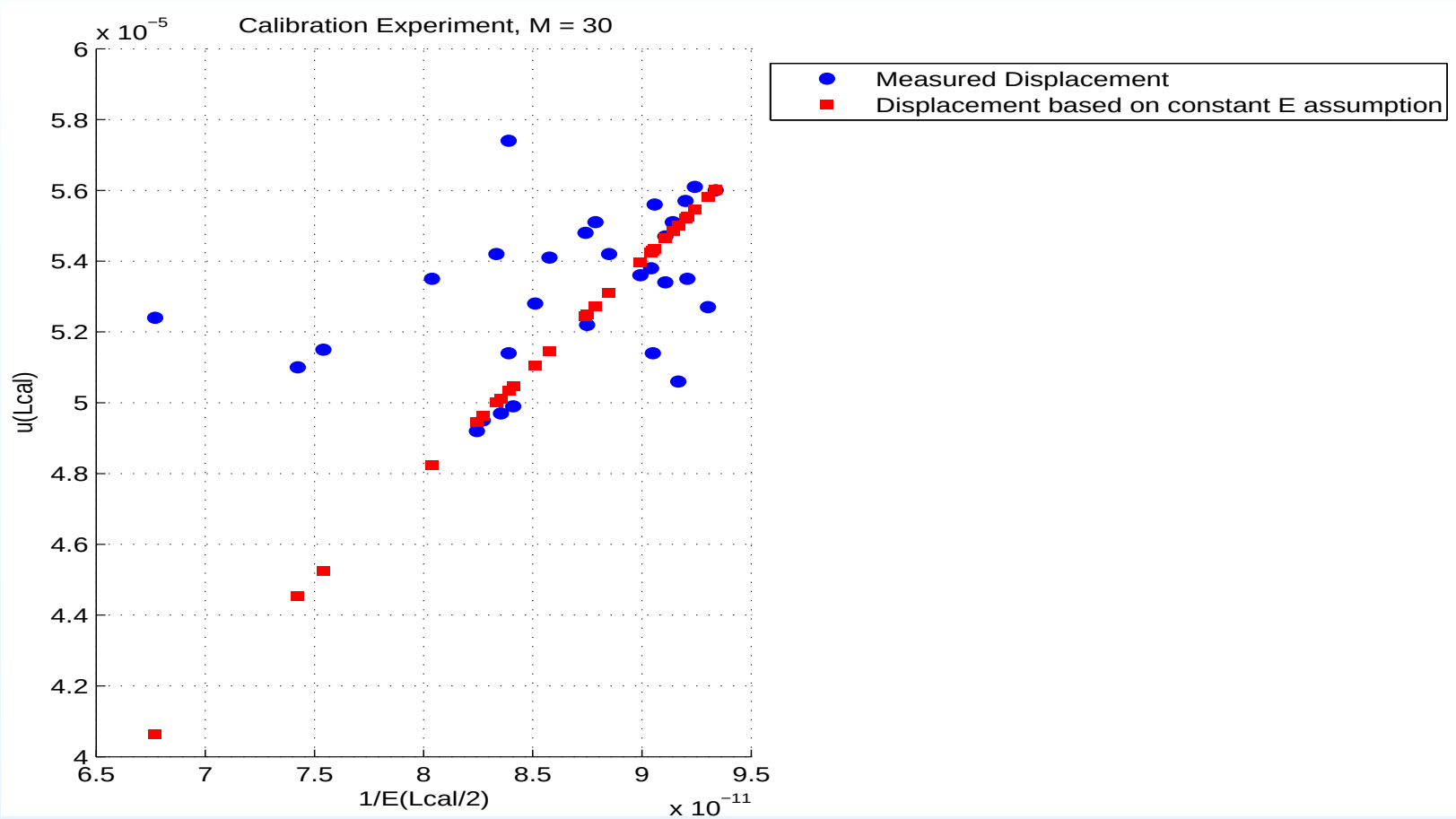
Amount of data available:

Case 1: **Small number of experiments**

Case 2: **Moderate number of experiments**

Case 3: **Larger number of experiments**

The homogeneous model doesn't reproduce correctly the given calibration data



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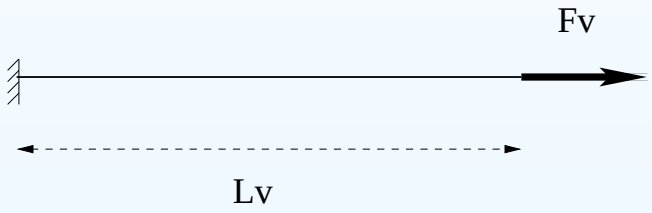
Goal **Validation of the probability field constructed in the calibration.**



elongation at the end of the bar

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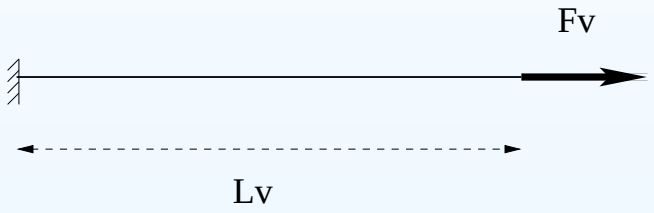


elongation at the end of the bar

Number of validation tests

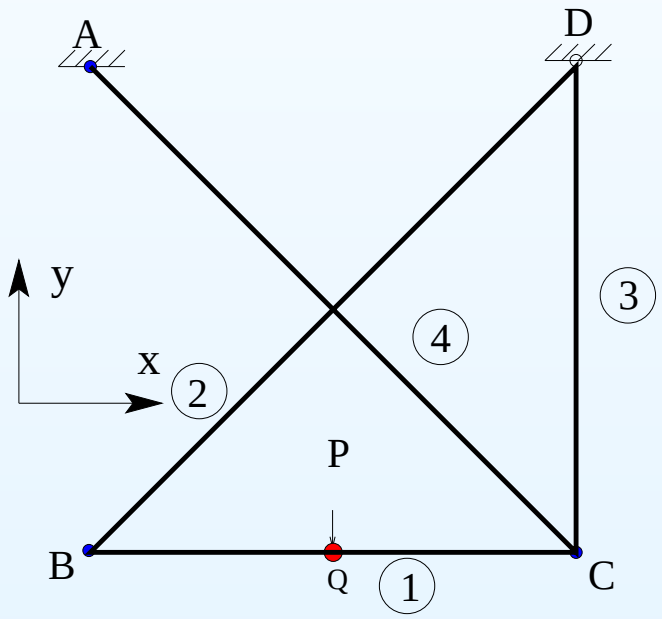
Case 1: (2)

Case 2: (4)

Case 3: (10)

Accreditation experiments

Goal **Validation of the model in an environment that has some similarities to the prediction.**



vertical displacement at midpoint of bar #1

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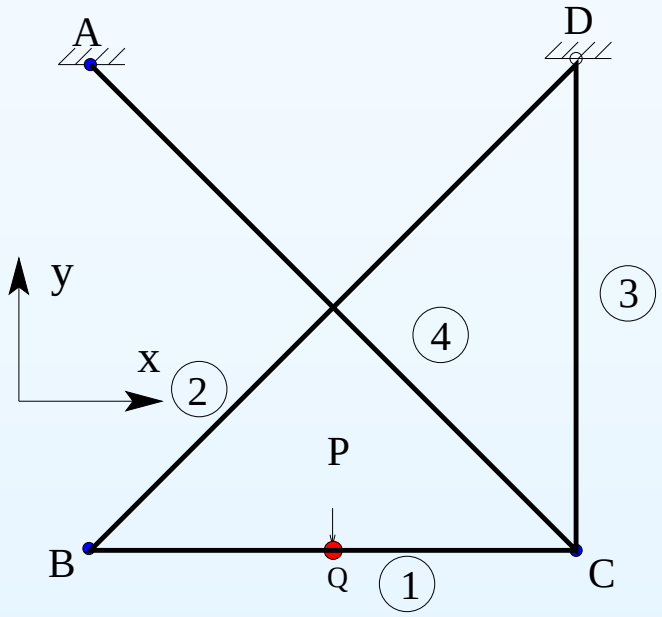

vertical displacement at midpoint of bar #1

Number of accreditation tests

Case 1: (1)

Case 2: (1)

Case 3: (2)

- **Difference with the prediction problem:**

  a) **different geometry**

  b) **different load**

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Prediction

Regulatory compliance: **likelihood of the event**

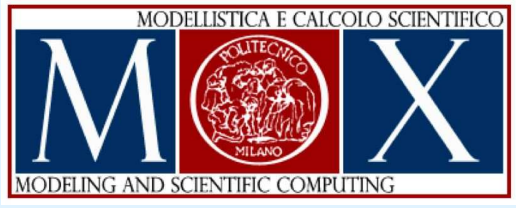
$$|w| \leq 3.0 \times 10^{-3} m$$

i.e.

$$P(|w| \leq 3.0 \times 10^{-3} m) = \alpha$$

and a quantitative statement on the number α.

Validation followup discussion



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WHAT IS THE PURPOSE OF THE COMPUTATION?

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Validation Criteria:

- ARE WE CHECKING IF THE MODEL IS CORRECT?
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- OR CHECKING INSTEAD IF THE MODEL IS SUFFICIENTLY ACCURATE FOR OUR PARTICULAR APPLICATION?

- IS IT AGREEABLE TO SAY THAT WE ARE INTERESTED IN A FINITE NUMBER OF QUANTITIES OF INTEREST DEPENDING ON A GIVEN APPLICATION (AND THE SET OF THOSE QUANTITIES IS WHAT WE CALL THE PREDICTION)?

Validation followup discussion

- VERIFICATION: REMEMBER THAT IT COMES BEFORE VALIDATION!

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- SHOULD THE VALIDATION METRIC BE RELATED AS MUCH AS POSSIBLE TO THE DESIRED PREDICTION? HOW CAN THIS GOAL BE ACHIEVED?

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Validation Criteria:

- SHOULD THE VALIDATION METRIC BE RELATED AS MUCH AS POSSIBLE TO THE DESIRED PREDICTION? HOW CAN THIS GOAL BE ACHIEVED?
- HOW MANY DIFFERENT EXPERIMENTAL SETUPS ARE NEEDED FOR THE VALIDATION/ACCREDITATION PROCESS? SOME COST/BENEFIT DISCUSSION SHOULD BE INCLUDED.

Validation followup discussion

- UNCERTAINTY QUANTIFICATION: WHICH ARE THE RIGHT TOOLS?